Cyclogenesis: Simulating Hurricanes and Tornadoes

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Fig. 1. Simulation of a hurricane forming over the Atlantic Ocean traveling through the Gulf of Mexico until it hits the US coast and starts decaying due to the lack of enough latent heat to sustain vorticity. The hurricane’s trajectory is similar to the one of the category-5-hurricane Katrina which particularly hit the city of New Orleans and its surrounding area in 2005 causing over 1300 fatalities and severe damage. The hurricane’s intensity is color-coded along the trajectory according to the Saffir–Simpson scale [Taylor et al. 2010] ranging from a tropical depression to a category-5-hurricane (see top right corner).

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Cyclones are large-scale phenomena that result from complex heat and water transfer processes in the atmosphere, as well as from the interaction of multiple hydrometeors, i.e., water and ice particles. When cyclones make landfall, they are considered natural disasters and spawn dread and awe alike. We propose a physically-based approach to describe the 3D development of cyclones in a visually convincing and physically plausible manner. Our approach allows us to capture large-scale heat and water continuity, turbulent microphysical dynamics of hydrometeors, and mesoscale cyclonic processes within the planetary boundary layer. Modeling these processes enables us to simulate multiple hurricane and tornado phenomena. We evaluate our simulations quantitatively by comparing to real data from storm soundings and observations of hurricane landfall from climatology research. Additionally, qualitative comparisons to previous methods are performed to validate the different parts of our scheme. In summary, our model simulates...
Cyclogenesis in a comprehensive way that allows us to interactively render animations of some of the most complex weather events.


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1 INTRODUCTION

Cyclogenesis denotes the formation or strengthening of a low-pressure area that favors the formation of tropical cyclones. Such tropical cyclones are rapidly rotating storm systems that feature a low-pressure center, a closed low-level atmospheric circulation, and powerful winds. These storms are organized in a spiral pattern of thunderstorms with heavy rain and squalls. Depending on where they form and how strong they are, these storms may be referred to as “hurricanes” or “typhoons”. Hurricanes are strong tropical cyclones that occur over the Atlantic or northeastern Pacific Ocean, while typhoons form in the northwestern Pacific Ocean.1 Given the significance of cyclones as natural disasters and downright terrifying phenomena, they have received a considerable amount of scientific interest in a wide range of different fields of research. In climatology or meteorology, research has focused on mesoscale simulations employing statistical as well as principled approaches, e.g., Cui and Caracoglia [2019]. These simulations usually emphasize specific aspects of cyclonic phenomena but do not describe them comprehensively within an integrated model that includes changes in the diurnal cycle, microphysical processes, and dynamically changing boundary conditions. Moreover, these models require, in general, the use of supercomputers and specialized hardware and software architectures [Orf 2019]. This makes a direct application of these methods for visual computing applications unfeasible. In graphics, several research works were proposed towards modeling cloud formations and other weather phenomena, e.g., Amador Herrera et al. [2021], but none of these approaches consider the additional turbulent-flow processes that can develop into a hurricane.

In this paper, we propose a physically-based approach to describe the formation of cyclones. Our method explicitly models the turbulent microphysics which forms the basis of cyclogenesis by coupling different interacting hydrometeors: Cloud water, ice, rain, snow, and graupel (i.e., precipitated ice). Additionally, we incorporate a two-fluid model for tornadogenesis to describe the emergent development of vortex tubes which may form as a consequence of cyclonic dynamics. Finally, we introduce a mathematical model that captures the large-scale transfer of heat and vapor between water bodies and the atmosphere, which leads to the formation of hurricanes.

Our key contributions are: (1) We propose a comprehensive physically-based scheme for computing the turbulent transport of heat and water in the atmosphere, which includes the multi-scale simulation of vortex phenomena as well as two-fluid coupling for interacting dust and debris; (2) We close our turbulent-flow equations by formulating extended eddy mixing microphysics, modeling the interactions between ice and water particles; (3) We address the dynamics of the emergence, development, and dissipation of cyclonic phenomena at different scales, enabling the visually realistic and physically plausible simulation of these extreme weather events, as demonstrated by multiple validation and comparison experiments.

2 RELATED WORK

The modeling and simulation of cyclone dynamics and turbulent weather phenomena is an ongoing research topic in different academic communities. While this spans a breadth of work that we cannot conclusively discuss here, we provide references to the modeling and simulation of local weather, physical studies of tornadic phenomena, and the simulation of cyclogenesis at the mesoscale. Outside of visual computing, there have been multiple studies on different aspects of turbulent phenomena. Cyclogenesis within a storm has been studied in detail by Klemp [1987], and Rotunno and Klemp [1985]. Development of convection and general circulation at thunderstorm boundaries has been investigated by Droegemeier and Wilhelmson [1985]. On another direction, the role of latent heat on generating and sustaining vortexes has been studied numerically by Gao et al. [2019] and experimentally by Sheets [1982]. The goal of our work is to provide a comprehensive framework for the interactive simulation of turbulent weather dynamics at both the mesoscale and the storm-scale. In the following paragraphs, we will discuss related work.

Weather Simulation. One of the first methods for simulating clouds based on the underlying atmospheric phenomena was introduced by Kajiya and Von Herzen [1984]. Several interactive cloud simulation approaches have been proposed that range from grid-based fluid solvers [Dobashi et al. 2000; Harris et al. 2003; Miyazaki et al. 2002, 2001; Overby et al. 2002], particle-based approaches [Goswami and Neyret 2017] and methods based on GPU-parallelization [Schalkwijk et al. 2015]. To also enable artistic control of modeling clouds, procedural techniques have been proposed [Webanck et al. 2018]. Due to the complexity of simulating physics various representations have been explored that enable the efficient simulation and modeling of clouds and weather [Bouthors and Neyret 2004; Dobashi 2002; Gardner 1985; Neyret 1997; Nishita et al. 1996] including position-based dynamics [Ferreira Barbosa et al. 2015] and layer-based approaches [Vimont et al. 2020]. A common approach is to use hierarchical and adaptive grid structures to simulate the fluid dynamics for clouds [Raateland et al. 2022]. Other methods focus on large-eddy phenomena [Griffith et al. 2009], rain [Garcia-Dorado et al. 2017] and snow [Gissler et al. 2020], supercells [Hädrich et al. 2020], and complex weatherscapes [Amador Herrera et al. 2021]. It has been recognized that vegetation can contribute to local weather variations resulting in diverse microclimates [Pahubicki et al. 2022]. Furthermore, wildfires generating flammagenitus clouds have been simulated [Hädrich et al. 2021]. Despite these advances, it is important to note that none of these approaches consider a turbulent scheme, so cyclonic phenomena can only be prescribed but not simulated from first-principles.

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1In the Indian Ocean, South Pacific, or South Atlantic, these storms are simply called “tropical cyclones”.

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Tornadic Phenomena. The work of Orf et al. [2017] achieved impressive results of tornadic phenomena within a supercell, including the formation, evolution and decay of multiple vortices. However, their scheme uses a supercomputer, billions of grid nodes and is far from being an interactive approach. A similar work [Orf 2019] uses grids of the order of trillions of nodes. The same holds true for other mesoscale frameworks where the focus is on processing precise physical data for comparison to storm soundings [Miglietta et al. 2017; Pilgju et al. 2019]. In the visual computing community, tornades have been simulated by Liu et al. [2006] whose model can generate visually convincing tornades but relies on prescribed rotational boundary conditions. Liu et al. [2007] improves the model by considering a reynolds averaging dynamics and coupling the tornado to destructive domains, but suffers from the same high dependence on boundary conditions. Also in these models the supercell is descriptively modeled and only the tornado (lower vortex tubes) is simulated emergently.

Cyclonic Modelling. Within the field of visual computing, cloud dynamics and weather phenomena in general have not been simulated on the hurricane scale. While the works of Hadrich et al. [2020] and Amador Herrera et al. [2021] do consider supercell formation and development, they only model the formation of a single cumulonimbus, and not a cluster of clouds that can evolve into a hurricane. Additionally, there is no work in the visual computing community that explicitly handles turbulence and cyclonic dynamics within a cloud scheme. In the atmospheric science community, multiple efforts have been made to study different aspects of cyclonic phenomena at the mesoscale from the stochastic generation and decay of hurricanes [Cui and Caracoglia 2019], the influence of latent heat in turbulent cumulus convection [Kuo 1965], dynamics of the energy budget within mesoscale storms [Peng and Kuo 1975], to the nonlinear dynamics of wind fields within typhoons [Vickery et al. 2000]. In the context of scientific visualizations, some works explore techniques for the interactive visual analysis of hurricane data [Doleisch et al. 2004], while other focus on developing educational visualizations of hurricanes [Luo et al. 2008]. Note, however, that the data for visualization in these works is not simulated, just taken a priori. In contrast to research in tornadic phenomena, works on turbulent dynamics associated to hurricanes tend to focus on analyzing specific aspects of cyclonic phenomena, as opposed to having a comprehensive framework for their simulation that includes diurnal cycles, ice-phase microphysics, and dynamic boundaries.

3 OVERVIEW

The principal motivation for our approach is to realistically model and simulate turbulent weather phenomena using a comprehensive scheme that can be used for graphics applications. This is a challenging task due to the complex interplay of heat and fluid dynamics within a turbulent wind field, as well as due to the presence of debris and mesoscale phenomena that determine the regional energy budget. We address these challenges by proposing an integrated physically-based model that targets the multi-scale simulation of turbulent heat and water transport in the atmosphere. As illustrated in Figure 2, our approach aims for a compromise between interactivity and physical complexity. Our framework empowers artists with interactive modeling control over a rich set of cyclonic phenomena.

We describe the state of the atmosphere using five main quantities: the wind velocity $u$, the velocity of dust in the air $u_d$, the amount of water in the atmosphere $q_a$ (in the form of vapor $q_v$, rain $q_r$, etc.), the temperature $\theta$ of humid air, and the turbulent energy $k$ which sustains turbulent motion. At its core, our cyclogenesis model consists on coupling these quantities to capture the interplay of turbulent heat and water continuity. In this sense, our model can be divided into (I) a subgrid-energy scheme that describes enhanced transport due to turbulence, closure equations that incorporate the associated (II) turbulent microphysics and (III) Reynolds-Averaged Navier Stokes’ dynamics, (IV) an extended Kesler-type approach to address turbulent heat and water dynamics, and (V) a two-fluid model that couples atmospheric and dust fields, as illustrated in Figure 3. Since hurricanes and tornadoes operate at different scales, we propose different set of equations (III.1) and (IV.1) for mesoscale cyclonic phenomena, which take into account the energy budget of water bodies, the axis-symmetrical nature of hurricanes, and the Coriolis effect.

4 METHODOLOGY

In this section, we provide an outline of our physics-based cyclogenesis model. It is a general and efficient scheme for the turbulent transport of heat and water in the atmosphere, including different hydrometeors or particles: vapor $q_v$, cloud water $q_w$, cloud ice $q_i$, rain $q_r$, snow $q_s$, and graupel $q_g$. For convenience, we use tensor notation for the derivation of our Reynolds-Averaged formulation. A brief explanation of this type of notation is presented in Appendix A.2. Additionally, a table of symbols, including the values used for our simulations, is provided in Appendix A.1.

4.1 Atmospheric Model

Our atmospheric scheme starts with a parameterization of the reference background atmosphere, as well as a thermodynamic model for the rising thermal of humid air.

4.1.1 Background Atmosphere. The surrounding dry air is parameterized in terms of its time-dependent temperature $T$ and pressure fields $p$ in space $x = (x, y, z)$. These fields can be streamed directly as input from real measurements (see Section 5.2); otherwise, we assume a temperature field that resembles the standard atmospheric conditions.
Amador Herrera, J. A. et al.

Fig. 3. Schematic representation of our cyclogenesis framework for turbulent heat and water continuity. The arrows indicate different inter-dependencies. Our procedure starts with (I) the computation of subgrid kinetic energy; then, we use this result to solve (II) the associated turbulent microphysics as well as (III) the turbulent fluid dynamics. Afterwards, we use both the velocity field and turbulent terms to compute (IV) the transport of atmospheric water content. Note, that our multi-scale scheme uses a different set of equations for steps (III.1) and (IV.1), which accounts for different cyclonic phenomena. Finally, for the tornado scheme, we use the computed velocity to solve (V) the equations of the coupled dust field.

4.1.2 Momentum Equations. The next step in our model consists in describing the turbulent motion of humid air. In particular, we propose a formulation based on the Reynolds-Averaged form of the Navier-Stokes equations (RANS) that can be easily coupled to microphysical phase changes in the atmosphere, e.g., cloud condensation, ice crystallization, etc. In the RANS formulation, it is assumed that the velocity field \( \mathbf{u}(x, t) \) can be decomposed into an average flow \( \bar{\mathbf{u}}(x) \) and a fluctuating term \( \mathbf{u}'(x, t) \), so that

\[
\mathbf{u}(x, t) = \bar{\mathbf{u}}(x) + \mathbf{u}'(x, t).
\]

Other fluid quantities (e.g., pressure, stress, etc.) are decomposed analogously. Then, taking the average value of this system, it is possible to derive a set of correspondent RANS equations describing the average flow of momentum [Alfonsi 2009], given in tensor form

\[
\rho \frac{\partial \bar{u}_i}{\partial t} + \rho \mathbf{u} \cdot \nabla \bar{u}_i + \frac{\partial \tau_{ij}}{\partial x_j} = \mathbf{f}_i,
\]

where \( \rho \) is the density of the fluid, \( \mathbf{f}_i \) is any external influence, and the total stress tensor \( \tilde{\tau}_{ij} \) can be expanded into

\[
\tilde{\tau}_{ij} = -\bar{p} \delta_{ij} + 2\mu \ddot{\mathbf{e}}_{ij} - \rho \mathbf{u}_i \mathbf{u}_j',
\]

with potential temperature \( \theta \) defined by \( T = \Pi \theta \).

\[\begin{align*}
\text{(I)} & \quad \frac{D\phi}{Dt} = \frac{\partial}{\partial x_j} \left( \nu M \frac{\partial \phi}{\partial x_j} \right) \\
\text{(II)} & \quad M_a = \sum_{j \neq a} M_{a,j} - \sum_{k \neq a} M_{a,k} \\
\text{(III)} & \quad \frac{\partial \bar{u}_i}{\partial t} + \mathbf{u} \cdot \nabla \bar{u}_i + \frac{\partial \tau_{ij}}{\partial x_j} = \mathbf{f}_i \\
\text{(IV)} & \quad \frac{D\phi}{Dt} = \bar{\rho} + D_i \\
\text{(V)} & \quad F_c = \bar{\rho} d - \mathbf{u}_v
\end{align*}\]
with pressure $\tilde{p}$, dynamic viscosity $\mu$, Reynolds stress tensor $\tilde{u}_i u_j$, and strain rate tensor $\tilde{S}_{ij}$ computed as

$$\tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

This system of equations can be closed using zero-, one-, and two-moment schemes [Kajishima and Taira 2016]. We propose a one-moment closure model that can also describe microphysical phase changes. First, we assume that vortex tubes evolve without being subject to complex nonlinear interactions, e.g., collisions between tornadoes or with the environment, so we compute the Reynolds stress tensor using the Boussinesq linear eddy-viscosity equation, which reads

$$\tilde{u}_i u_j = -2\nu_T \tilde{S}_{ij} + \frac{2}{3} k \delta_{ij},$$

with turbulent viscosity $\nu_T$ and turbulent kinetic energy $k$. Then, we modify the turbulent energy scheme of [Mellor and Yamada 1974] to account for ice-phase transitions, such that the turbulent kinetic energy is given by

$$\frac{Dk}{Dt} = g v_M \frac{\partial \theta}{\partial x} + v_M \frac{\partial \theta_T}{\partial x_i} + \frac{\partial}{\partial x_i} \left( \nu_T \frac{\partial \theta}{\partial x_i} \right) - C e \sqrt{k},$$

(5)

where $D/Dt = \partial/\partial t + \tilde{u}_i \partial/\partial x_i$ is the material derivative, $v_M$ represents eddy mixing, $s_k$ is the subgrid turbulence scale, $C e = 0.2$ is a ventilation coefficient, and, in our specific formulation, $\theta_T = \hat{\theta}_x + \hat{\theta}_i + \hat{\theta}_e$ is the total mixing ratio of water content in the form of ice, warm cloud, and vapor. The turbulent viscosity and eddy mixing terms are updated as

$$\nu_T = 0.2 s_k \sqrt{k},$$

(6)

$$v_M = 3 C e \nu_T,$$

where $C e$ is an eddy coefficient acting as a coupling strength constant between microphysical phenomena and turbulent kinetic energy. Once we have a closed system for turbulent dynamics, we introduce the density-driven buoyancy force via the $f_i$ vector in Eq. (4). Specifically, atmospheric buoyancy is computed following Archimedes’ principle, and reads

$$B(x,t) = g \left[ \frac{\theta}{\theta_0} - 1 + 0.61 \hat{\theta}_x + \sum_a \hat{\theta}_a \right].$$

(7)

Finally, the Coriolis effect is incorporated directly by an additional term $K_{cor} = \epsilon_{ijk} \alpha_{cor} j \hat{u}_k$, where $\alpha_{cor}$ is a vector representing the angular velocity with respect to each coordinate axis.

4.1.3 Turbulent Microphysics. The formation of different cloud configurations in the atmosphere is heavily influenced by microphysical processes [Amador Herrera et al. 2021]. Since we want to model not only the turbulent flow of wind, but also the formation, maturation, and dissipation of cloud cyclone structures, we incorporate a microphysics scheme that can describe water phase changes under turbulent motion. Our microphysics model, illustrated schematically in Figure 4, consists of an extra system of coupled transport equations that account for the diverse physical interactions between different hydrometeors. In general, the microphysics parametrization $M_{a,b}(T, \rho, q_a, q_b)$ of a species $a$ transitioning into $b$ (e.g., snow melting into rain $M_{s,r}$) will be a function of local temperature, pressure, and current mass fractions of $a$ and $b$. The total rate of phase change of a hydrometeor $a$ is then expressed as

$$M_a = \sum_{j \neq a} M_{j,a} - \sum_{k \neq a} M_{a,k},$$

(8)

where the sources $M_{j,a}$ and sinks $M_{a,k}$ include transitions via condensation, deposition, sublimation, evaporation, etc. The explicit parametrization of each process is taken directly from Amador Herrera et al. [2021]. We further modify this equation to include additional eddy mixing of temperature and hydrometeors. Then, our general expression for the dynamics of potential temperature $\theta$ and mixing ratios $\tilde{q}_a$ is given by

$$\frac{D\tilde{q}_a}{Dt} = M_{\tilde{q}_a} + D_{\tilde{q}_a},$$

(9)

where $\phi$ represents a temperature or mixing ratio field, $M_{\phi}$ is the usual microphysical term, and $D_{\phi}$ is a turbulence mixing variable. In particular, $M_{\theta}$ represents the local change in temperature due to water phase transitions, so it is given by

$$M_{\theta} = \sum_a \frac{L_e}{c_p} X_a.$$

(10)

where the sum runs over all phase transitions, $L_e$ is the latent heat of the transition, and $X_a$ the mass fraction of each hydrometeor. From mixing length theory, it is known that the turbulent mixing of fields acts as an enhanced diffusion process scaled by the turbulent kinetic energy. For this reason, we use the prognosis equations

$$D_{\tilde{q}_a} = \frac{\partial}{\partial x_j} \left( v_M \frac{\partial \tilde{q}_a}{\partial x_j} \right),$$

(11)

where the eddy mixing term $v_M$ explicitly models the influence of turbulent kinetic energy on enhanced microphysical diffusion.

4.1.4 Vorticity Dynamics. In vector form, the first expression in Eq. (4) can be written as

$$\frac{\partial \tilde{u}}{\partial t} + (\tilde{u} \cdot \nabla) \tilde{u} = \frac{1}{\rho} \tilde{\nabla} k + F + \frac{2\mu}{\rho} \tilde{\nabla}^2 \tilde{u} - \nabla \tilde{\rho},$$

where $\tilde{\nabla}$ is the spatial gradient operator. This equation, along with the equations for pressure and density, constitutes a system of conservation equations for the velocity field in a rotating frame.

Fig. 4. Illustration of the turbulent microphysics model for the transport between different cloud and precipitation types. The scheme includes both microphysical matter transport $M_{\phi}$ as well as turbulence mixing $D_{\phi}$ per phase transition.
where we have grouped the turbulent and Coriolis terms in
\[ F = 2\nu_T \nabla^2 \boldsymbol{u} - \frac{2}{3} \nabla k + \alpha_{\text{cor}} \times \boldsymbol{u}. \]

Computing the curl on both sides of Eq. (4.1.4), we get the dynamic equations for turbulent vorticity \( \omega = (\nabla \times \boldsymbol{u}) \), written as
\[
\frac{D\tilde{\omega}_h}{Dt} = \tilde{\omega} \cdot \nabla \tilde{u}_h + \tilde{\omega} \frac{\partial \tilde{u}_h}{\partial z} + F'_z, \quad (12)
\]
\[
\frac{D\tilde{\omega}_h}{Dt} = \tilde{\omega} \cdot \nabla \tilde{u}_h + \nabla \times (\tilde{b} \tilde{k}) + F'_h,
\]
where the subscripts \( z \) and \( h \) indicate the vertical and horizontal components of a vector, respectively, and \( F' = \nabla \times F \) is the curl of all fields included in \( F \). We see from the right-hand-side of Eq. (12) that, for a velocity field that starts with no prescribed rotation, the process of cyclogenesis is started by the tilting \( \omega \cdot \nabla \tilde{u}_h \) and mixing \( F'_z \) parameters, which transform horizontal vortex lines into the vertical vortex tubes that form a tornado. The production of vertical vorticity, then, depends crucially on both turbulence mixing fields and environmental wind-shear.

4.2 Two-Fluid Tornadic Parametrization

The process of tornadogenesis involves the development of several vortex tubes in the wind field, even at zones where condensation is not present. These tubes can be visualized and analyzed directly by plotting \( \omega \) [Orf 2019]. However, this approach does not capture the expected visuals of a tornado made of dust and debris that got inside the non-condensed vortexes. Instead, we use an approach close to the Two-Fluid Model (TFM) of Liu et al. [2006], in which a dust field is coupled to \( \tilde{u} \) to enhance tornado visualization by incorporating the movement of debris particles. Note, however, that in TFM only the bottom tornadic region is considered, i.e., there is no mesoscale or cloud simulation. Since the dust field \( \tilde{u}_\text{dust} \) does not involve any microphysical change, but constant particles of radius \( r_{\text{dust}} \), we propose a zero-moment RANS, which reads
\[
\rho_{\text{dust}} \frac{D\tilde{u}_{\text{dust}}}{Dt} = \nu_{\text{dust}} \nabla^2 \tilde{u}_{\text{dust}} - \nabla \tilde{p}_{\text{dust}}
+ \nabla \cdot \tilde{\tau}_{\text{dust}} + F_{\text{dust}} + m_{\text{dust}} g,
\]
\[ \nabla \cdot \tilde{u}_{\text{dust}} = 0, \]
with debris mass \( m_{\text{dust}} \), viscosity \( \nu_{\text{dust}} \), zero-moment Reynolds stress \( \tilde{\tau}_{\text{dust}} \), pressure \( \tilde{p}_{\text{dust}} \), density \( \rho_{\text{dust}} \), gravitational constant \( g \), and external forces \( F_{\text{dust}} \). In particular, \( F_{\text{dust}} \) contains the coupling term \( F_c \) between air and dust. The interaction is parameterized as
\[
F_c = \frac{\rho_{\text{dust}} \tilde{u}_{\text{dust}} - \tilde{u}}{C_{\text{dust}}} , \quad (14)
\]
where the scalar \( C_{\text{dust}} \) is computed as
\[
C_{\text{dust}} = \frac{m_{\text{dust}}}{3 \pi r_{\text{dust}} \nu_{\text{dust}}} \left( 1 + \frac{Re}{60} + \frac{Re/4}{1 + \sqrt{Re}} \right),
\]
\[
Re = \rho_{\text{dust}} r_{\text{dust}} \sqrt{\frac{\nu_{\text{dust}}^2}{\nu_{\text{dust}}^2}}.
\]
Finally, the zero-moment Reynolds stress is computed as
\[
\tilde{\tau}_{\text{dust}} = \rho_{\text{dust}} \tilde{u} \cdot \frac{\partial \tilde{u}}{\partial z} \frac{z^2}{c_k^2},
\]
where \( z \) is the altitude, and \( c_k = 0.4 \) is the Von Kármán constant.

4.3 Cyclone Scheme

As opposed to tornadic phenomena, the primary energy source for the genesis and maintenance of tropical cyclones is the latent heat released by condensation of oceanic water [Holland and Merrill 1984]. In this sense, the large-scale motion in a hurricane consists of an axisymmetric forced circulation driven by the heat released in convective cells. Based on this, we propose a numerical model for the treatment of mesoscale cyclonic dynamics based on the assumptions of axial symmetry and hydrostatic balance, similar to planetary boundary layer (PBL) approaches [Moeng 1984]. In cylindrical coordinates \((r, \gamma, z)\), we express the velocity using its radial, tangential, and vertical components \( \tilde{u}_{\text{rad}}, \tilde{u}_{\text{tan}}, \text{and } \tilde{u}_{\text{vert}} \), respectively, so that the RANS equations for a meridional plane are given by
\[
\frac{\partial \tilde{u}_{\text{tan}}}{\partial t} + \tilde{u}_{\text{rad}} \frac{\partial \tilde{u}_{\text{tan}}}{\partial r} + \tilde{u}_{\text{vert}} \frac{\partial \tilde{u}_{\text{tan}}}{\partial z} + \tilde{u}_{\text{vert}} \frac{\partial \tilde{u}_{\text{tan}}}{\partial r} = \frac{Q}{\Pi c_p} + \nu_T \nabla^2 \tilde{\theta} + \frac{g h}{\Pi c_p} \frac{\partial \tilde{\theta}}{\partial p}, \quad (15)
\]
\[
\tilde{u}_{\text{tan}} \left( \alpha_{\text{cor}} + \frac{\partial \tilde{u}_{\text{tan}}}{\partial r} \right) = \frac{\partial \Phi}{\partial r}, \quad (16)
\]
where \( \Phi \) the geopotential associated to the earth’s gravitational field. Additionally, the conservation of energy in cylindrical coordinates is expressed as
\[
\frac{\partial \tilde{\theta}}{\partial t} + \tilde{u}_{\text{rad}} \frac{\partial \tilde{\theta}}{\partial r} + \tilde{u}_{\text{vert}} \frac{\partial \tilde{\theta}}{\partial z} = M_{\theta} + D_{\theta} + \frac{\partial m}{\partial p} , \quad (18)
\]
where \( m \) represents the vertical flux of moisture. We describe the mean stress tensor, as well as the fluxes of momentum, sensible heat, and moisture using the usual prognosis equations for tropical cyclones [Kepert 2010], given by
\[
\tilde{r} = \rho_d \bar{\| \tilde{u} \| \tilde{u}_{\text{tan}}},
\]
\[
h = c p \rho_d \bar{\| \tilde{u} \| T},
\]
\[
m = \rho_d \bar{\| \tilde{u} \| q_v},
\]
\[
Q = c p \left( \bar{\Pi} \tilde{\theta} - T \right).
\]
Finally, to evaluate the derivatives with respect to pressure, we use the chain rule and the radial pressure gradient
\[
\frac{\partial \tilde{p}}{\partial r} = B \left( \frac{R_{\text{max}}}{r} \right)^B \exp \left[ - \left( \frac{R_{\text{max}}}{r} \right)^B \right],
\]
where \( B = 0.5 \) is Holland’s radial pressure profile parameter [1980], and \( R_{\text{max}} \) is the maximum radius variable (with approximative values of 8 km for small storms, and up to 150 km for large hurricanes).
ALGORITHM 1: Cyclogenesis Algorithm.

Input: Current system state \((\bar{u}, \bar{u}_d, k, \theta, q_a)\).

Output: Updated system state.

Procedure:

\(T, p \leftarrow\) Update background temperature and pressure using either input data or Eqs. (1) and (2).

\(k \leftarrow\) Advect and diffuse turbulent energy \(k\).

\(v_p \leftarrow 0.2 q_a \bar{v}^2\) Update turbulent viscosity

\(v_M \leftarrow 3C_e v_p\) Update eddy mixing

\(X_a \leftarrow \varphi / q_a\) Convert mixing ratio to mass fraction

\(\tilde{B} \leftarrow\) Compute buoyancy force according to Eq. (7)

\(F_c \leftarrow\) Update wind-debris interaction term following Eq. (14)

\(w \leftarrow\) Advect vorticity as an independent field, according to Eq. (12).

\(\tilde{u} \leftarrow \tilde{u} + B + \Delta x N \times w\) Apply buoyancy and vorticity confinement

\(\bar{u} \leftarrow\) Advection, diffuse, and pressure project the wind field

\(\bar{u}_d \leftarrow\) Advection, diffuse, and pressure project debris field

\(M_0 \leftarrow \sum_{r} \bar{b}_r N_a q_p\) Compute temperature change

\(\bar{\varphi} \leftarrow \bar{\varphi} + M_0 + D_0\) Turbulent Microphysics

\(\bar{\varphi} \leftarrow\) Advect and diffuse additional fields

end

ALGORITHM 2: Mesoscale Cyclone Algorithm.

Input: Current system state \((\bar{u}, \bar{u}_d, k, \theta, q_a)\).

Output: Updated system state.

Procedure:

\(T, p \leftarrow\) Update background temperature and pressure using either input data or Eqs. (1) and (2).

\(k \leftarrow\) Advect and diffuse turbulent energy \(k\).

\(v_p \leftarrow 0.2 q_a \bar{v}^2\) Update turbulent viscosity

\(v_M \leftarrow 3C_e v_p\) Update eddy mixing

\(\tilde{r}, h, m, Q \leftarrow\) Update moisture, momentum, and sensible heat fluxes using Eq. (19).

\(X_0 \leftarrow \varphi / q_a\) Convert mixing ratio to mass fraction

\(B \leftarrow\) Compute buoyancy force according to Eq. (7).

\(w \leftarrow\) Advect vorticity as an independent field, according to Eq. (12).

\(\tilde{u} \leftarrow \tilde{u} + B + \Delta x N \times w\) Apply buoyancy and vorticity confinement

\(\bar{u} \leftarrow\) Advection and diffuse the wind field following Eq. (15)

\(\bar{\varphi} \leftarrow\) Advection and diffuse potential temperature following Eq. (16)

\(\tilde{\varphi} \leftarrow \tilde{\varphi} + M_0 + D_0\) Turbulent Microphysics

\(\tilde{\varphi} \leftarrow\) Advect and diffuse additional fields according to Eq. (18)

end

5 ALGORITHMS

In the following, we provide details for the numerical integration procedure, including the setup of our discretization scheme, boundary conditions, and solver. Our tornadic and cyclone frameworks are summarized in Algorithm 1 and Algorithm 2, respectively.

5.1 Numerical Integration

The turbulent heat, water, and fluid dynamic models described in the previous sections provide the basis for the implementation of our cyclone simulation framework.

5.1.1 Mesh Structure. We set up a staggered 3D voxel space using an uniform grid scale \(\Delta x\), and set \(s_k = (\Delta x \Delta y \Delta z)^{1/3} = \Delta x\). We store the current state of our atmospheric system in this grid: velocity fields \(\bar{u}\) and \(\bar{u}_d\) are stored at the faces, while turbulent kinetic energy \(k\), mixing ratios \(q_a\), potential temperature \(\theta\), and vorticity \(\tilde{\omega}\) are stored at the center. Additionally, we discretize all the derivative operators using centered finite differences, as shown in Appendix A.4. To account for ground conditions, e.g., temperature, water content, and pressure, we also include a 2D uniform grid using the same scale \(\Delta x\), as well as a height map \(H : (x_1, x_2) \rightarrow H(x_1, x_2)\) such that the ground mesh is embedded in the 3D space as \(\delta \Omega_{\text{bottom}} = (x_1, x_2, H(x_1, x_2)) \in \Omega\). To update the terms involving material derivatives, we use a semi-Lagrangian scheme, while quantities that are updated directly, e.g., mixing coefficients and atmospheric profiles, are computed on the fly.

5.1.2 Implementation. First, the subgrid kinetic energy \(k\) is advected while no-slip conditions are set at the bottom, and free-slip boundaries at the ceiling and walls. After updating the eddy coefficients, heat transfer and microphysical terms, we compute the relevant fields that control the forces acting on the velocity field (e.g., buoyancy for tornadic phenomena and also sensible heat fluxes in the case of mesoscale cyclones). Then, the wind field is advected followed by the integration of viscosity-related effects by solving the corresponding diffusion process [Stam 1999]. For the advection process, we employ the same boundary conditions as those for subgrid kinetic energy. Finally, when updating the potential temperature and hydrometeor particles through their material derivatives, we set \(\theta\) to the ambient temperature at the boundary; additionally, periodic boundary conditions are used for vapor \(q_v\), and all the other hydrometeors \(q_{\phi}\) are set identically to zero at the sides. For our hurricane scheme, we set \(\theta\) to zero at the origin \(r = 0\), as well as \(\partial \theta / \partial r = \delta \theta_{\phi} / \partial r = 0\). The surrounding \(r_{\text{max}}\) conditions are set the same as in rectangular coordinates. Finally, for the debris field \(\bar{u}_d\) we set Dirichlet boundaries at the bottom (the amount of dust in the ground), periodic conditions at the sides, and free-slip at the top.

5.1.3 Vorticity Confinement. We implement vorticity confinement to mitigate the effects of numerical dissipation. However, we found that by updating the vorticity for the confinement process using Eq. (12) instead of the usual numerical cross product, the quantitative results of angular velocity for hurricanes were closer to real measurements (Figures 23 and 24) without requiring fine-tuning. Specifically, we treat the vorticity as an additional field that is added to the wind velocity via confinement with a force \(f \propto \nabla N \times w\), where \(N = \nabla \eta /|\nabla \eta|\) and \(\eta = |w|\). Note that, since we only use the vorticity to reduce numerical dissipation and not to reconstruct the velocity field, the momentum equation is only advected one time, and we do not need to solve additional stream functions.

5.1.4 Numerical Solver. In both of our cyclone generation schemes, we use a one-moment turbulence approach based on computing the contribution of subgrid turbulent energy to the average turbulent atmospheric flow. Additionally, our microphysics scheme also computes averaged quantities over ensembles of particles that would be otherwise resolved at a subgrid scale. Given this multi-scale nature of our framework, a natural path to solve our RANS formulation in a highly efficient manner is to use a state-of-the-art algebraic multi-grid (AMG) solver. Specifically, we use the AMGCL solver proposed by Demidov [2019], which is able to efficiently solve large sparse linear systems.
5.2 Dynamic Boundary Conditions

In general, cyclonic schemes for the efficient visual simulation of tornadic phenomena heavily rely on the boundary conditions of \( \mathbf{u} \) for the generation of vorticity. In consequence, it is customary in these methods to adopt fixed no-slip and Dirichlet boundaries at the sides, top and bottom of the mesh to generate an \textit{a priori} rotational field [Ding 2005]. This approach has been useful to simulate different tornadic phenomena, e.g., the simulation of destructive tornadoes by Liu et al. [2007], but it restricts both the domains and cyclonic effects that can be modeled. In contrast, our physically inspired model can produce vorticity even when rotation is not prescribed at the boundary (b) and also with dynamic conditions like inlet gravity waves (c).

We use four maps to encode, respectively, surface temperature map, surface temperature map, and heat capacity map.

The development and subsequent decay of cyclonic phenomena depend not only on the local turbulent transport of heat and water, but also on global weather conditions, which we incorporate as the boundary values of incoming winds, temperature, pressure, and humidity in the environment. To simulate global changes in the environment surrounding the cyclone, we enable the dynamical change of our boundary conditions. For incoming winds, the time-dependence is directly encoded in \( \mathbf{W} = \mathbf{W}(t) \), e.g., for periodic wave forms as discussed before, or simply by reducing the wind shear intensity over time. Additionally, we use dynamic boundary values when streaming real-data measurements into our framework. For instance, for the simulation of specific hurricane events like Katrina and Diane, we directly stream as boundary conditions real measurements of temperature, pressure, and humidity from the center of the cyclone is used for simulations of the hurricane eye, while (right) temperature profiles are streamed as ground boundaries (by fixing \( T_G(x, y) = T_{\text{profile}} - \Gamma_{\text{profile}} \)) for simulating the whole cyclone structure. Additional time-dependent profiles for humidity and pressure can be streamed into our framework analogously. These plots are reproduced using data from Kossin and Eastin [2001].

6 RESULTS

In this section, we present a variety of results simulated with our C++/CUDA framework implemented as described in the previous section. Table 1 provides an overview of the different scenes presented throughout this section, including relevant parameters. Computation times listed in Table 1 are measured using an NVIDIA® GTX 2080 Ti, with double precision floating point arithmetic.

6.1 Vortex Generation

First, we validate the different components of our \textit{cyclogenesis} framework by comparing the generation of vortex phenomena with different approaches.
The emergent vortex formation is only possible in our extended moment closure system which, in turn, eliminates the artifacts at the boundaries of the cluster.

Table 1. Overview of the relevant parameters used in the scenes presented in this paper. For all scenes, a constant time step size of $\Delta t = 5$ min is used. Parameters are listed in $[T_{ij}] = 1^\circ C$, $[\overline{\rho} y_r] = 1$ kg kg$^{-1}$, and $[W] = 1$ km hr$^{-1}$. Identical parameter values $\rho_d = 2500$ kg m$^{-3}$, $c_p = 0.4$, $\Gamma = -6.5$ K/km, and $z_1 = 8$ km are used in all simulations. Resolution (R) and runtime (T) in seconds per frame are listed.

<table>
<thead>
<tr>
<th>Fig.</th>
<th>Scene</th>
<th>$T_{ij}$</th>
<th>$\overline{\rho} y_r$</th>
<th>$W$</th>
<th>$C_p$</th>
<th>R</th>
<th>T</th>
</tr>
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<td>150</td>
<td>0.8</td>
<td>800</td>
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<td>30</td>
<td>0.4</td>
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<tr>
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<td>Turbulence Comparison</td>
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<td>30</td>
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<td>512</td>
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<td>512</td>
<td>0.11</td>
</tr>
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<td>–</td>
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<td>0.6</td>
<td>512</td>
<td>0.14</td>
</tr>
</tbody>
</table>

6.1.1 Turbulent Microphysics. As demonstrated in Figure 8, the weatherscapes framework of Amador Herrera et al. [2021] is not able to generate vortexes unless explicit rotational boundaries are specified. Moreover, even with prescribed rotation, the tornado generated by weatherscapes is less visually appealing because it lacks subgrid details caused by both tilting and turbulence effects, as shown in Figure 10. These experiments show the advantages of explicitly modeling turbulent microphysics phenomena for simulating cyclonic motion in the atmosphere.

6.1.2 One-Moment RANS. Previous tornadic schemes [Ding 2005; Liu et al. 2007] are able to capture interesting vortex dynamics of tornadoes using a zero-moment RANS formulation for turbulent dynamics. However, these schemes model only the lower part of the tornadic region, and do not take into account the associated rotating supercell. In Figure 10, we show the simulation of a rotating cluster of clouds using a zero-moment closure and our model. The inclusion of turbulent microphysics enables us to formulate a one-moment closure system which, in turn, eliminates the artifacts at the boundaries of the cluster.

6.1.3 Force-Driven Approach. Outside of weather phenomena, many efforts have been made to incorporate turbulent dynamics into visual simulations. In particular, the MacCormack scheme [Selle et al. 2008] is close to our framework as it is built on top of the usual semi-Lagrangian solver for smoke motion while incorporating a second-moment closure for turbulence. In Figure 11, we compare the simulation of a tornado with our framework and a force-driven approach using the MacCormack scheme. In order to ensure a fair comparison, we use the same boundary conditions at the sides and top of the domain, while the bottom boundary in the force-driven case is set to a vertical shear $W k$ around the inlet to simulate the buoyancy in the atmosphere. While the force-driven simulation is able to produce interesting subgrid turbulence, as in Liu et al. [2007], it does not capture the global shape of the tornado, including the parent supercell, because the formation of these atmospheric structures is governed not only by wind motion but also by phase transitions at the microphysical level (e.g., different cloud types vary in their particle compositions [Amador Herrera et al. 2021]). Moreover, the process of cyclogenesis is not reproduced correctly by the force-driven approach because it generates the vortex from bottom to top following the wind field. In contrast, the tornado in our model grows from top to bottom, following the direction of condensation, even though the wind field is moving upwards. Analogously, the force-driven approach can be used to simulate a mesoscale vortex but, as demonstrated in Figure 12, it is not able to reproduce the eye structure of a hurricane. In Figure 13, we show quantitative measurements of these experiments. In summary, we observe that a turbulent force-driven model can simulate wind fields similar to those encountered in atmospheric cyclones but the lack of microphysical phase transitions impacts...
the final global geometry of the vortex (Figure 12) and the vertical cloud fraction profiles (Figure 13).

6.2 Vortex Control

Our physically based model enables us to easily control the simulation of vortex dynamics via a lightweight canonical parameter set. In particular, while the temperature and water content determine the final cloud geometry, the number and strength of vortex tubes can be easily controlled via the turbulence or mixing coupling constant $C_e$, and the boundary wind shear magnitude $||W||$. More intense winds generate greater angular velocities, while higher values of the turbulence constant increase the number of vortexes.

6.3 Tornado Types

As demonstrated in Figure 15, our framework is able to simulate different types of tornadoes caused by the complex interplay of a turbulent atmosphere, microphysical phenomena, and wind shears: (a) Using low eddy mixing coefficients $C_e$ as well as low shear $||W||$ forms relatively uniform and dense cold clouds that trigger small vortex tubes beneath them. Some of these tubes carry the necessary pressure and temperature conditions to generate condensation along their path, which in turn creates small rope tornadoes. (b, c) Increasing the shear while maintaining relatively low mixing coefficients introduce enough turbulence into the system to sustain a greater vortex tube of condensation and debris that is captured from the ground, forming a cone or funnel tornado. (d, e) When we further increase $C_e$, the vorticity field generates multiple small vertical vortex tubes that are combined into a single large eddy of condensation and precipitation around it in the form of rain. These wider eddies are called wedge tornadoes, and are usually within the EF4-EF5 category on the Enhanced Fujita scale. (f) Decreasing
the temperature while increasing the amount of humidity $q_v$ in the atmosphere enables us to simulate the base of mixed-phase cumulonimbus clouds where vorticity is present at the rotating base but pressure and temperature conditions do not allow condensation at such altitudes, which results in a bowl-shaped tornado. (g) Increasing the mixing coefficients in this conditions enables some vorticity tubes to travel down the atmosphere even when they transport no condensation. When such a tube reaches the ground, it may trap debris around it, which allows to visualize part of the vortex connecting the ground and the base of the cloud. This phenomena is commonly known as landspout. (h) Increasing both wind-shear and eddy mixing enables us to simulate strong turbulence dynamics that can sustain multiple-vortex tornadoes at the base of the cumulonimbus.

An additional benefit of a physically-based model over turbulent force-driven schemes is that we can compute the value of diverse atmospheric quantities and compare against real data. For instance, in Figure 16 we show the correlation between the Bulk Richardson Number (BRi) – a ratio of buoyancy to vertical shear – and the Storm Relative Environmnetal Helicity (SREH) – a measure of the streamwise vorticity within the inflow environment of a convective storm – with the intensity of a tornado in the Enhanced Fujita scale, and how our computations compare with the data measured by Colquhoun and Riley [1996] and later extended by Anderson-Frey et al. [2019]. For the comparisons, we simulated thirty funnel tornado events per Enhanced Fujita category, while varying the velocities within the velocity range of real events. Our simulation results match the observed correlations in real tornadoes. The computations of BRi and SREH are detailed in Appendix A.3.

6.4 Dust Devils

Similar to landspouts, dust devils are ground-based whirlwinds that spin upward, in contrast to tornadoes that spin downward from the base of a cloud. The main difference between these two types of vortexes is that, while landspouts are formed below a supercell and travel upwards following a non-condensed vortex tube, dust devils typically form on clear days when warm air rises into cooler air above, which generates a vortex spinning from the ground upward. Our vorticity dynamics model can simulate different whirlwind phenomena not only at different scales but also made up of different types of matter: Condensed water in the atmosphere, debris trapped in vortex tubes, and dust. In particular, dust devils can be easily generated by our framework since we already simulate the turbulent flow of heat in the atmosphere. This is demonstrated in Figure 17, where we input wind-shear over a dusty terrain, generating a well-formed and short-lived whirlwind.

6.5 Tornado Life Cycle

We reproduce the process of tornadogenesis and tornado decay, as shown in Figure 21, by first setting up a mature thunderstorm, and, afterwards, let a wind-shear evolve so that the rotating cumulonimbus generates a tornado on its base. Once the non-local shear at the boundary ceases to exist, the tornado decays and gradually dissipates. Additionally, we are able to simulate the transition between tornado types, as shown in Figure 22, where a wedge tornado evolves into a funnel structure until it disappears since it lacks the energy to sustain the vortex field.

6.6 Mesoscale Cyclones

To evaluate the mesoscale counterpart of our scheme, we carry out different validation experiments of hurricane phenomena.

6.6.1 Gravity Waves. We simulate the formation of large-scale vortex phenomena generated by gravity waves in the atmosphere, as shown in Figure 18. Our framework is able to capture the initial stages of hurricane formation, including the transition from local disturbances to a sustained rotational field. As discussed before, our one-moment RANS formulation is able to handle dynamic boundaries with incoming impulses.

6.6.2 Hurricane Eye. Next, we focus on the very center of the hurricane, which enables us to visualize the so-called hurricane eye. This region of the cyclone, shown in Figure 19, using atmospheric profiles from hurricane Diane (which occurred in August 1995 and has been the first Atlantic hurricane to cause more than an estimated one billion in damage), consists of a roughly circular area surrounded by the eyewall, which is a ring of clouds that rotate about the vortex. Inside the eye, the pressure can be as low as to stop condensation, which enables the formation of relatively clear skies inside of it. A photographic comparison is also shown in Figure 20. Quantitatively, we compute the angular velocity from the eye to the radius of maximum wind (RMW) along the pressure isosurface of 850 mb (around 1.5 km from the ground). We make two measurements: At $t = 0.5$ h and 3 h, respectively. As shown in Figure 24, our framework is able to reproduce the observed regimes of angular velocity for this hurricane event [Kossin and Eastin 2001]. Note, that we used multiple runs to account for the uncertainty in the measured profiles for this event. Moreover, in Figure 23, we compare one test run for this experiment using our direct vorticity update and the usual cross-product vorticity confinement as in Amador Herrera et al. [2021]. Our scheme produces less numerical dissipation, which is crucial for quantitative comparisons against observed dissipation.

6.6.3 Sea Temperature. Finally, we analyzed the effect of surface sea temperature (SST) on the decay of hurricanes after landfall, i.e., when they hit land. For this, we generated four hurricane events with surface temperatures $T_G$ from 26.85° C to 29.85° C, and measured their velocity profiles while traveling towards the land and decaying after hitting the coast. The results, depicted in Figure 25, show that, for greater levels of SST, the decay rate is slower, which means the hurricane is sustained for a longer time in land even after latent heat from the ocean has stopped fueling the vorticity of the cyclone. The values of hurricane decay obtained using our framework match the behaviour and value range of state-of-the-art data analysis [Fogarty et al. 2006; Li and Chakraborty 2020].

6.7 Weather Nowcasting

Finally, we streamed a full set of diverse atmospheric measurements (shear, temperature, pressure, and humidity) from weather radar...
Fig. 15. Variations of different types of tornadoes simulated using our framework (top) and corresponding photo comparisons (bottom): Rope (a), funnel (b, c), wedge (d, e), bowl-shape (f), landspout (g), multi-vortex (h).

Fig. 16. We simulate thirty funnel tornado events per Fujita category, with varying velocities to match the velocity range of observed events. The correlations between tornado intensity with the Bulk Richardson Number (BRi) and Storm Relative Enviromental Helicity (SREH) match the real measurements of Anderson-Frey et al. [2019].

Fig. 17. Our vorticity dynamics scheme is able to simulate general vortex phenomena at different scales and within diverse contexts: Inside a supercell, from debris that got inside a vortex tube, and dust in the ground. In this simulation, a dust devil vortex (time evolution from left to right, top to bottom) is generated by the encounter of warm wind-shear with cool air in the atmosphere.

7 DISCUSSION

Our physically-based scheme describing the process of cyclogenesis enables the realistic simulation of multiple atmospheric vortex phenomena at different spatial and temporal scales. Note, that there are

\footnotesize{Data taken from the www.ventusky.com service.}
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Fig. 18. Time evolution (from left to right) of an inlet gravity wave, from the top corner of the domain, which later evolves into a hurricane formation.

Fig. 19. One characteristic feature of mesoscale cyclones is the hurricane eye at the center of the vortex. Inside of this region, relatively weak winds and low pressure generate relatively clear skies.

Fig. 20. Hurricane eye simulated using our hurricane module (left), and corresponding photo comparisons (right).

Fig. 21. Temporal evolution (from left to right) of the formation and subsequent decay of a cone tornado formed at the base of a rotating supercell.

many complex atmospheric models tailored for highly specialized computations of very specific weather phenomena and, while built based on principles from atmospheric science and fluid dynamics,
Limitations and Future Work. Our framework can be extended in various directions. First, multiple eyewalls may emerge within strong tropical cyclones, and under certain circumstances, the inner eyewall can even be replaced by an outer eyewall. This replacement can play an important role in the development of atmospheric conditions that could be explored and incorporated into our framework, e.g., vegetation feedback and lightning strikes. Finally, our cyclone scheme is limited to hurricanes with a single main rotation axis. It would be interesting to explore a more general formulation that would allow the simulation of mesoscale phenomena with multiple cyclones, e.g., the Fujiwhara Effect of colliding hurricanes.

8 CONCLUSION

We have proposed a novel physically-based model for the efficient and comprehensive simulation of turbulent phenomena in the atmosphere. Our approach explicitly models and integrates vorticity and turbulent multi-physics by parametrizing and coupling the underlying microphysical, fluid, and heat dynamics processes that generate vortexes at different scales. We have shown that our framework is capable of simulating the dynamic emergence, development, and dissipation of cyclonic phenomena, including different types of tornadoes and hurricane events. Furthermore, we have validated our scheme by performing multiple comparison experiments against both state-of-the-art and real data from storm soundings and radar technologies. The results of these validation experiments show that our scheme improves the visual simulation of cyclones and is able to reproduce similar results as those obtained from highly specialized atmospheric models, while still being a comprehensive and efficient framework that can be used for generating immersive virtual environments.

Future work in this direction includes exploring more complex hurricane dynamics that take into account the interaction of multiple eyewalls at the center of the hurricane, and the implementation of a more complex coupling between vortexes and the surrounding terrain. In this sense, it would be interesting to incorporate destructible domains that affect the dynamics of cyclones. Finally, there are multiple physical processes like vegetation feedback, lightning, etc., that we currently do not take into account but that play an important role in cyclonic phenomena. Integrating these effects into our framework would enable the simulation of more complex scenes.

A APPENDIX

A.1 Table of Symbols

Table 2 provides an overview of parameters and variables, including its numerical values and units, in order of appearance.

A.2 Tensor Notation

Using this notation, a tensor is represented by its indices, with the rule that a sum is performed for every repeating index. For instance, the dot product of two vectors $\mathbf{a}$ and $\mathbf{b}$ is written as

$$a_i b_i = a_1 b_1 + a_2 b_2 + a_3 b_3.$$  

Other useful operations include the cross product, written as $c_i = \epsilon_{ijk} a_j b_k$, and matrix multiplication, which in this notation reads $C_{ij} = A_{ik} B_{kj}$, for matrices $A$ and $B$. In the context of differential operators, the same rules apply for vectors of differentials. For instance, the divergence of a vector function $\mathbf{u}$ reads $\nabla \cdot \mathbf{u} = \partial u_i / \partial x_i$. 

\[ \nabla \cdot \mathbf{u} = \partial u_i / \partial x_i. \]
Table 2. List of different symbols (in order of appearance) used in our model and their typical values for simulation. Note that quantities without a shown value correspond to dynamic parameters computed at simulation time.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
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<td>x,y,z</td>
<td>Cartesian Coordinates</td>
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</tr>
<tr>
<td>r</td>
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<td>Wind Velocity</td>
<td>-</td>
<td>m s⁻¹</td>
</tr>
<tr>
<td>v</td>
<td>Dynamic Viscosity</td>
<td>1.865</td>
<td>kg m⁻¹ s⁻¹</td>
</tr>
<tr>
<td>𝜈ₑ</td>
<td>Turbulent Viscosity</td>
<td>-</td>
<td>m² s⁻¹</td>
</tr>
<tr>
<td>k</td>
<td>Turbulent Kinetic Energy</td>
<td>-</td>
<td>m² s⁻¹</td>
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<tr>
<td>t</td>
<td>Subgrid Scale</td>
<td>-</td>
<td>m</td>
</tr>
<tr>
<td>c_v</td>
<td>Ventilation Coefficient</td>
<td>0.2</td>
<td>1</td>
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<tr>
<td>𝜈ₑ</td>
<td>Eddy Mixing</td>
<td>-</td>
<td>m² s⁻¹</td>
</tr>
<tr>
<td>c_s</td>
<td>Mixing Constant</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>Buoyancy Acceleration</td>
<td>-</td>
<td>m s⁻²</td>
</tr>
<tr>
<td>M_T</td>
<td>Microphysical Phase Change</td>
<td>-</td>
<td>kg g⁻¹ s⁻¹</td>
</tr>
<tr>
<td>D_T</td>
<td>Microphysical Turbulent Term</td>
<td>-</td>
<td>kg g⁻¹ s⁻¹</td>
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<tr>
<td>L_T</td>
<td>Latent Heat Index: fusion L, etc.</td>
<td>-</td>
<td>J kg⁻¹</td>
</tr>
<tr>
<td>Q_L</td>
<td>Latent Heat of Vaporization</td>
<td>2260</td>
<td>J kg⁻¹</td>
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<tr>
<td>Q_F</td>
<td>Latent Heat of Fusion</td>
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<td>J kg⁻¹</td>
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<tr>
<td>Q_S</td>
<td>Latent Heat of Sublimation</td>
<td>2838</td>
<td>J kg⁻¹</td>
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<tr>
<td>X₀</td>
<td>Mass Fraction of a</td>
<td>-</td>
<td>kg g⁻¹</td>
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<tr>
<td>ρ_dust</td>
<td>Dust Density</td>
<td>490</td>
<td>kg m⁻³</td>
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<tr>
<td>m_dust</td>
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<td>7.5 x 10⁻⁵</td>
<td>kg</td>
</tr>
<tr>
<td>r_dust</td>
<td>Dust Radius</td>
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<td>ρ_v</td>
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<td>1.285</td>
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<tr>
<td>α_k</td>
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<tr>
<td>s, r, ζ</td>
<td>Cylindrical Coordinates</td>
<td>-</td>
<td>m</td>
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<tr>
<td>Φ</td>
<td>Earth’s Geopotential</td>
<td>-</td>
<td>m² s⁻²</td>
</tr>
<tr>
<td>h</td>
<td>Flux of Sensible Heat</td>
<td>-</td>
<td>J m⁻² s⁻¹</td>
</tr>
<tr>
<td>m</td>
<td>Vertical Flux of Moisture</td>
<td>-</td>
<td>kg m⁻² s⁻¹</td>
</tr>
<tr>
<td>Q</td>
<td>Non-adiabatic Heating</td>
<td>-</td>
<td>J m⁻³</td>
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<td>Holland’s Radial Parameter</td>
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<td>R_max</td>
<td>Maximum Storm Radius</td>
<td>150</td>
<td>km</td>
</tr>
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</table>

### A.3 Thermodynamic Variables

For comparing against real data, we compute two additional thermodynamic variables of storms. Specifically, the Bulk Richardson Number (BRi) and the Storm Relative Environmental Helicity (SREH). For the BRi, we simply use

$$ BR_i = \frac{(g/T_0) \theta_0 dz}{(DU)^2 + (AV)^2}, $$

with gravity acceleration g, absolute temperature T, potential temperature θ₀, layer size dz, and wind shear in the layer DU and AV. Following real measurements, we computed the BRi at the 1 km layer. For the environmental helicity, we compute

$$ SREH = \int_0^h \left[ k \frac{\partial |W|}{\partial z} \right] \cdot (W - \bar{u}) \, dz, $$

with vertical unit vector k, wind shear W, and storm velocity u. Again, according to measurements, we integrate at an altitude of h = 3 km.

### A.4 Discrete Differential Operators

The numerical discretization of our cyclogenesis model is performed using a staggered grid where the velocity fields of wind and dust, u = (u, v, w) and u_d = (u_d, v_d, w_d), respectively, are stored at the faces of each voxel, while all the other scalar quantities s, as well as the vorticity ω, are stored at the center (red point).

kinetic energy, mixing ratios, and potential temperature) as well as the vorticity ω are stored at the center of the voxels, as demonstrated in Figure 27. Using this grid, we compute the discrete version of our model using centered finite differences. The divergence of the velocity fields is computed as

$$ (\nabla \cdot \mathbf{u})_{i,j,k} = \frac{u_{i+1,j,k} - u_{i,j,k}}{\Delta x} + \frac{u_{i,j+1,k} - u_{i,j,k}}{\Delta y} + \frac{u_{i,j,k+1} - u_{i,j,k}}{\Delta z}, $$

where Δx, Δy, and Δz are the mesh step sizes in each dimension. Additionally, the gradient of scalar quantities is given by

$$ \frac{\partial s}{\partial x}_{i+1/2,j,k} = \frac{s_{i+1,j,k} - s_{i,j,k}}{\Delta x}, $$

$$ \frac{\partial s}{\partial y}_{i,j+1/2,k} = \frac{s_{i,j+1,k} - s_{i,j,k}}{\Delta y}, $$

$$ \frac{\partial s}{\partial z}_{i,j,k+1/2} = \frac{s_{i,j,k+1} - s_{i,j,k}}{\Delta z}. $$

Finally, for the discrete curl of the velocity we compute

$$ (\nabla \times \mathbf{u})_{i,j,k} = \left( \frac{w_{i,j,k+1} - w_{i,j,k-1}}{2\Delta z} - \frac{w_{i,j,k+1} - w_{i,j,k-1}}{2\Delta z} \right), $$

where the velocity components at the center are computed by averaging the values at the faces, such that

$$ u_{i,j,k} = \left( \frac{u_{i-1/2,j,k} + u_{i+1/2,j,k}}{2} \right), $$

$$ v_{i,j,k} = \left( \frac{v_{i,j-1/2,k} + v_{i,j+1/2,k}}{2} \right), $$

$$ w_{i,j,k} = \left( \frac{w_{i,j,k-1/2} + w_{i,j,k+1/2}}{2} \right). $$

### REFERENCES
